The answers to your new problems No. 12 - 17

12. Is it possible to expect further increase of MTD advantages as contrasted to turbo and other codes? Can they create the software MTD versions of algorithms with decoding operations number per bit in 1000 times smaller, than for other codes?

Now effective at a large noise level software turbo decoders usually demand less than 10'000 operations per bit. If they admit, that multithreshold decoder (MTD) will be built with complexity, which one will appear in 1000 times smaller, than for turbo codes, we get, that from MTD it is required to execute less than 10 operations per bit. Some turbo decoders execute essentially smaller number of operations then 10^4. Thus, MTD would execute in this case in general only a few calculations. Clearly, it is not real.

And if to compare capabilities MTD and turbo codes in the realm of such a noise, where MTD still successfully work, but the turbo decoders have not so good characteristics of the energy efficiency, it is possible, that for certain coding parameters the concatenated versions MTD really will give concerning other methods considerable advantage on throughput. The concatenated versions of software MTD versions of a similar type are built and now are widely investigated .

However the 100-multiple advantage MTD on number of operations before other algorithms so is extremely great. It already can be never notably reduced.

Quite real advantage about 1000 times on throughput for hardware versions MTD decoders was reviewed in the answer to the previous problem.

13. Where is in essence the difference between MTD and the customary threshold Massey's decoder?

It is only in that the solutions of all threshold switches (TS) in MTD are stored in the new differential memory register of the decoder and will be used then at the TS's decisions on following iterations of error correction. It is enough to prove **strictly completely unique property of the MTD**

decisions moving to the decisions of optimum (with the total search!) decoder (OD). Any other algorithm of error correction has no such extremely useful property, which one becomes specially relevant in iterative decoding circuits, to which ones MTD belongs too. Moreover, MTD complexity - linear one from length of a code **n,** and, with very small factor near **n**.

As it is very important, we shall remind, that as moving to the solution of the optimum decoder (OD) we call a property MTD, according to which at all changes of decoding symbols (may be non-binary) the distance of the current MTD solution from the received message strictly decreases. Moreover, having achieved the OD decision, MTD does not leave it. It is possible to call this a very useful MTD property as a stability of the MTD decisions concerning the OD decisions.

It is useful to mark once more that in MTD the special codes with a very low level of an error propagation (EP) effect should be applied only. This effect comprehensively was theoretically reviewed in a number of publications on MTD. Some of them are on our web-site dedicated to the multithreshold algorithm: **www.mtdbest.iki.rssi.ru** .

14. Is it possible to create MTD for non-binary channels?

The whole theory of MTD algorithms for non-binary codes and channels was written and published more than 20 years ago. The estimations of the characteristics were constructed and the conforming codes permitting to achieve high performances of decoding have been found. Certainly, the comprehensive simulation of algorithm work in channels with a large noise level was performed also. Non-binary MTD (further - QMTD) are so easy in implementation, as well as customary binary multithreshold decoders.

The estimations of **QMTD** simulation results testify, that these decoders also are very simple algorithmically and have the characteristics, which ones **are much better, than for codes of Read-Solomon (RS).**

It is no wonder, as the decisions of QMTD also strictly move to the decision of non-binary optimum (certainly, with total search!) decoder (OD), and used in QMTD codes, in opposite to RS codes, can have enough large length, that even more increases the efficiency of our multithreshold algorithm and simultaneously strongly simplifies it.

Let's remark, that the unsatisfactory situation in total with a problem of simple and effective decoding in coding engineering becomes already completely intolerant by consideration of decoding problem for non-binary codes. This unexpected thought becomes more understandable for you if to mark, that, for example, in a non-binary channel with the basis **q**=256

complexity of full-scale Viterbi algorithm (VA) will have the order $\mathbf{q}^{K-1}$, where **K** - a code length. And it means, that even at **К**=3 and rather low VA efficiency for such a short code and non-binary channel its complexity at many decimal powers exceeds complexity **N** of the standard binary VA with **К**=7: $\mathbf{N \sim 3*2^{K-1}}$. So it follows that in a **q**-ary channel any real VA actually does not exist at all. **There are only two decoders: for RS-codes and MTD possible for implementation in non-binary channels.** But MTD is much more easier to realize, than error correcting algorithms for RS-codes. Simultaneously with this fact MTD is much more effective, because the RS-codes have strongly restricted length, and the codes for MTD can be a very lengthy ones. More over the complexity MTD does not depend on a code length at all. Moreover, by selection of a lengthy code for QMTD it appears even a little more simple.

The copies of the new articles about QMTD also are placed on our site: **www.mtdbest.iki.rssi.ru** .

15. How to design MTD and to select codes for it?

The MTD projecting - composite process, which one consists in the analysis and implementation of all issues of Requirement Specifications (RS), which ones are reviewed in a problem № 3 (in the first packet of the answers on problems of our readers). We have improved a little this RS by results of discussions on our site and with the decoder customers. Its last modification is submitted on a new educational page of our site. In order to evaluate correctly and to take into account in such a project all capabilities of codes and MTD algorithm, it is possible to use the characteristics of this method in our reference book on coding: **"Noiseproof Coding. Methods and Algorithms",** which one has already left to the public in the publishing house "The Hot Line - Telecom" in Moscow in 2004.

All data about publishing house (web-site **www.techbook.ru**), where you can purchase the reference book, and about a capability of acquisition of the reference book in other places it is also possible to find on our web-site: **www.mtdbest.iki.rssi.ru** . Certainly, you can purchase this reference book in the Internet or in bookshops of Moscow, St.-Petersburg and in other cities. It is possible also to find out about current availability of the book in Internets-shops on a site **www.findbook.ru** or in searching system **www.yandex.ru** with two first words of book name writing in frame and with making mark to search in Market.

To facilitate to you very hard work to choose of a decoding method we also suggest you our **"The Simulator of the Digital Channel"** with the most broad set of the best decoding algorithms, known now all over the world. In this sense our simulator realized, for example, on a high-speed notebook, has no even approximate clones among any systems of simulation of decoding algorithms. "T**he Simulator**" - most unique device and test-bench for decoders of different types. Number of decoding methods, which characteristics they can see and evaluate with their help, is continuously increasing. Already now its application will allow you to keep track of a global level of error correcting algorithms, and also to forecast it for many years forward.

The detail description of **"the Simulator"** is placed on our web-site **www.mtdbest.iki.rssi.ru** in section of news. As soon as possible we shall distribute free of charge demoversion of the Simulator with all main capabilities of its full-scale version. ***As soon as possible this demoversion of the Simulator will be located and on our web-site also.***

Soon we shall offer also soft-hardware complex for check MTD algorithms, designed for PLIS and other hardware.

Let us continue the main theme.

On independent mastering by technology of MTD decoders designing it is necessary to expend definite time. It is necessary also to have thus three program complexes: 1) for codes selection, 2) for MTD simulation, and also 3) - for optimization of its parameters. These program complexes are permanently developed by their authors.

For your work on creation of the soft or hardware MTD versions were maximum successful and allowed for all last achievements in the field of researches and designing of this algorithm, **address also directly to the developers of MTD algorithm.**

16. If in the project, considered for us, rather low-speed channels do not need a high rate of decoding, whether it is necessary even in this case to realize the MTD method? May be, it is necessary to apply any other algorithms?

Your concrete problem is directly connected to a philosophy of applied researches in coding theory.

The occurrence turbo of codes in 1993 became a revolution in coding due to two causes.

At first, it was connected with that the advance in coding engineering after appearance of the concatenated scheme which Viterbi algorithm (VA) and codes of Read-Solomon (RS) in 80-th years has almost stopped. And - for a long time!

(Certainly, the authors of our site **www.mtdbest.iki.rssi.ru** and algorithm MTD regret, that to the western specialists the numerous articles about algorithms MTD and book "Computer Networks" of publishing house "Nauka" ("Science"), Moscow, of 1981 are practically obscure, in which the fundamental theory of this method was stated. These articles demonstrated continuous advance in researches on subjects of multithreshold algorithms and in 80-th years.)

Secondly, directly in the first publication about turbo codes their authors for the first time have declared in essence as a new purpose of applied researches just the main problem of the coding theory: **simple decoding near channel capacity.** Moreover, authors of the first article about turbo code demonstrated almost actual capability for to reach this purpose.

Two circumstances mentioned above also become the cause of avalanche growth of the publications about turbo and many similar them algorithms. It was the new epoch in researches on codes. Certainly, more in-depth analysis was then fulfilled of this new situations in coding theory and in adjacent applied realms. It was found out, that actually in coding huge quantity of unsolved technological problems are and in turbo decoding, as in a well known TV-serial, "everything is still starting". Up to the final result - really simple and effective decoder, - as well as "a decade of turbo codes" has shown was a very far.

But 10 years of turbo codes development have passed now. Whether it is possible to make any generalizations? Yes, it is already possible particularly. Let's consider them.

Initially turbo decoder work near channel capacity demanded up to 10'000 operations per bit. And then, and now it was very much, especially for high-speed channels. Then many thousands explorers have managed considerably to simplify both turbo decoders, and very many other algorithms, which beginning of the development was initialized by turbo codes. However in most cases together with simplification of algorithms the characteristics of their efficiency also have worsened a little.

It would seem, it was necessary to take into account advantage in stating of new aims due to turbo codes and look for other methods to solve the main code problem near channel capacity. But these really new methods were absent.

Certainly, the explosion of concern to coding has recovered many earlier ideas in this area, in particular, LDPC codes and some other interesting code designs. All of them were connected to the different useful approaches, which ones, however, on an accuracy of their metric estimations selection

could not even come closer to MTD algorithm. It also resulted to unfairly high complexity of their implementation. So it was not possible at all to reach a MTD level by yardsticks of implementation complexity!

Thus, turbo decoders have given greatest acceleration in the coding theory development. They have indicated the worthy purposes and have demonstrated a reality of their achievement. But the "force" method of a decoding problem solution in turbo philosophy has not solved and principally could not solve problem of turbo decoding complexity, though the growth of throughput of element base of micro electronics was during this decade rather considerable. One of the causes of this failure is that the transmission rates during this decade have grown by very high races too. Therefore it is doubtless, that overall application of turbo codes does not seem realistic. The turbo decoding is too complex. And it is, probably, for a long time, if not for ever.

So on a background of this foregone crisis turbo subjects the outlook for MTD algorithms are looked absolutely differently.

At close capabilities on efficiency the MTD throughput, as shown in the answers to the previous problems, is much more higher, than for turbo decoders. Let's point out, that we say about a difference not in percentage or at all about advantage on rate efficiency in some times. **MTD advantage on decoding speed (number of operations) is amounting some decimal powers.**

By the way, the program for MTD may be placed on single page of the text, for example, in the language C++.

If the channel speed does not exceed 1 ÷ 3 Mb/s, hardware MTD realization are completely unnecessary. Moreover, the bound of microprocessor software decoder versions preference as contrasted to hardware will move to more high processing speeds. And the very large reserve software MTD on throughput in this case can be used for a designed communication network additional capabilities on reliability, synchronization, veracity and so on. **It also is the answer to your problem "what to do".** Always only the best!

And when the problem of coding for high-velocity channels is esteemed, for the benefit of MTD the actual capability of creation of hardware decoders of such a class, which one will be at some parameters of code systems **faster than other ones approximately at 3 decimal power!** The causes of this MTD advantage we have already considered earlier too. **This is a reality so!**

Therefore a problem on application of this or that coding system should always be decided uniquely: for any communications systems it is necessary to use only most effective for the current moment coding methods with minimum computing load at decoding.

The cause of it, we shall repeat, is simple: after the first coding system you will need to make other, then third and so on. If your second system was not the best by yardstick "complexity - efficiency", when you will begin to create a third system with the extremely possible characteristics, you will not have useful experience of the second system for new project at all.

17. If in our concrete project it is necessary to realize coding system with small decision marking delay, what can be a procedure selection of truly good method of decoding?

Your problem extends that formulation of a problem of decoder selection even more, relatively the previous answer to problems. The problem, actually put in your question, extends up to following: as far as the known algorithms are flexible in relation to different bottlenecks of decoding? Or if in one case the large productivity is most relevant, i. e. number of operations of decoding, in another case - small spreading of frequency band, and in the third - the small decision delay of the decoder, how is it necessary to select methods at this general formulation of a selection and implementation problem of an error correction system?

Let's begin with that quantity of substantially effective and simultaneously well constructed methods is very insignificant. There are turbo codes, VA and cascade circuits with VA, also some LDPC sorts of codes and, at last, MTD and special circuits with it.

So the small delay, about which one you ask, is required in low-speed channels, when even some superfluous milliseconds of decision making are critical for the project as a whole. If the delay is a very small one, at first it is necessary to compare it with spherical packaging bound (again!) and to see, whether these characteristics were theoretically possible at least.

The start impression thus is, that this situation corresponds always to necessity of AV application. However it is necessary to remember, that VA is a total searching algorithm. As it is known, in 70 years of the past century the decoder creators very fast have managed to reach such a level, that the standard for communications systems became VA with the code length K=7. After 30 years of the VA existence with an extremely rapid progress of element basis site authors only recently consulted development on creation the VA for a code with code generator length K=11. And, as it is known, characteristics of VA with K=11 are not much better, than for K=7. But VA complicating thus is rather considerable. The cause of it is again the same one - exponential growth of its complexity! So a rapid progress in coding with VA is not real. But MTD with possibilities of VA with K=15 already exists for a long time.

And turbo codes fall into a class of circuits with usually large delay also. So MTD and at small delay practically has no any real rival.

More over, it is always easier to create microprogram MTD decoder implementation and to use a little more composite logically decision making rule. It demands a very small number of second operation works. But the characteristics MTD, high enough initially, thus are follow-up improved.

The examples of submitted above reasoning about interchanging of parameters demand total generalization. It can be made thus. Between delay of decision making **L**, used memory **M**, code length **n** and throughput **N** in MTD wide parameter exchange is easy achievable. The listed above parameters can be varied in miscellaneous combinations more than in ten times. Any other methods of decoding with so broad permissible exchange ratio of key factors of efficiency and complexity in the present moment are not known. It is possible to find a capability of a so broad variation of the mentioned above parameters, and also for others, which ones were shown in RS for code systems (answer to a problem No. 3 and a new educational page of our web-site with new version of RS). It also allows to create on the MTD basis unique special error correction circuits, which ones, remaining in the class of MTD procedures, follow-up improve their very high characteristics.

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Conclusions. Any other methods of decoding do not allow so easily to manipulate practically all parameters of algorithms, as it is possible in a case of MTD. It allows to multithreshold decoders to benefit practically all actually objectively organized championships of algorithms on parameter set of efficiency, complexity and throughput.

Have you any new problems?
We shall try to answer very fast.