Introduction

The rapid growth of data processing, the development of digital broadcast systems and computing networks form very high demands to minimize errors in the used discrete information. Transformation of all types of creation, storage, usage and data transmission as well as broadcasting facilities to digital methods, what is happening now around the world, further increases the importance of high-quality digital stream transmission. Successful work of these systems is possible only in the presence of a special effective equipment that allows you to guarantee reliable information transmission. The most important contribution in improvement the reliability of the digital data exchange is introduced by the theory of noise-resistant coding, which creates new methods of error protection, based on the usage of error-correcting codes.

The possibility of using certain error correcting algorithms in systems without feedback is determined by highly strict requirements for these algorithms, for example, the number of operations in the case of their software implementation or in size, noise immunity, performance, power consumption and throughput of specialized LSI devises. A large number of regularly published monographs devoted to various aspects of the noise-proof coding theory, and dozens of international conferences on this subject, organized every year around the world, testifies to the great complexity and extreme urgency of the problem of effective decoding.

This book is devoted to a new stage of noise-resistant coding theory optimization development and created on its based error correction methods of digital data on the iterative majoritarian algorithms decoding basis, which recently again attracted the attention of specialists in the field of communication systems. The new state of coding theory is based on Optimization Theory (OT) technologies and ideology as well as achievement in all major types channels with independent distortions high noise immunity level in the realm close to the Shannon's bound, i.e. simple high-reliable decoding in such channels at code rate R very close to the channel capacity C, $R \lesssim C$.

We emphasize that for the implementation of high-performance algorithms near the Shannon's bound, it requires only long codes and the complexity of decoding, which should increase with the length of the code no more than linearly. These are MTD decoders and other methods based on them, presented in this book. Algorithms MTD achieve decisions of optimal (with total searching!) decoder (OD) with linear implementation complexity with their codes length. These decoders deleted absolutely from further competition of algorithms all procedures based on methods of algebraic theory codes, i.e. BCH and Reed - Solomon (RS) codes, as well as sequential procedures for convolutional codes. Apparently, it is the same applies to some recent "new" achievements of the theory encoding - polar and other codes, if they will not present, finally, enough meaningful results on real complexity and the effectiveness of

their proposed decoding algorithms. It further enhances the importance of OT achievements.

The majoritarian decoders has been discussed previously both in the classical monographs of L.F. Borodin [7], J.L. Massey [8], V.D. Kolesnik and E.T. Mironchikov [9], L.M. Fink [62] and the books [10-12], published later. A significant advantage of threshold (majority) decoding was the possibility of effective multiple improvement of these decoder's decisions, which was proved, in particular, in 1981 in the collective monograph published in the "Science" publishing house [2] on the example of systematic convolutional codes.

This book is devoted to the presentation various new aspects of the Optimization Theory (OT) of error-correcting coding and summarizing results of the researches of multithreshold decoders for binary and nonbinary codes used to transmit messages through channels with errors and erasures. Particular attention is paid to solving the problem of minimizing the volume of calculations while maintaining the highest possible energetic efficiency of encoding and small complexity of decoding.

The main objectives of the undertaken theoretical and experimental researches presented in this monograph are:

- theoretical justification of the methods of multithreshold decoding (MTD) linear codes comparable with the best known algorithms efficiency;
- analysis of special coding methods using paradigms OT and MTD, effectively operating near channel capacity.

The monograph consists of six chapters.

The first chapter introduces the basic concepts and definitions used in the following sections. In addition, there are basic information about the usual threshold decoders (TD) for the binary and symbolic codes.

The second chapter formulates the basic principles of multithreshold decoding for binary symmetric channels, and proves the aspiration of this decoder decisions to the decisions of the optimal decoder. This property of the MTD algorithm is a basic mean for implementation searching the global optimization of the functionals with a large number of variables for the particular case of discrete mathematical spaces.

Then this principle of increasing the likelihood of the MTD decisions is generalized to Gaussian channels when using "soft" modems, nonsystematic and nonbinary codes, and also on channels with erasures. At the end of the chapter, the limiting possibilities of various MTD types and the problem of errors propagation in majoritarian decoding, which solution allowed many times to improve the MTD characteristics and other related them methods.

In the third chapter on the basis of the recent publications the most important results on algorithms OT are presented, including the specific characteristics of these algorithms in direct close proximity to the Shannon's bound. The materials of this chapter have a great independent value, as all sections of this chapter are the finished published works and are characterized by very high information density of the stated material. Therefore, with the deep ideological connection of this chapter with the

content of the rest of the monograph all its sections are completely autonomous as well as separate the lists of used literature of sections-articles in each of them. We believe that it will be convenient for readers, who would, without reference to its other chapters or different publications, fully understand the content of these articles and paragraphs. References to literary sources, of course, beyond the third chapter are end-to-end for the entire book that well corresponds to the established traditions.

The fourth chapter describes the results obtained by taking into account the new paradigms of OT, which further improve the efficiency of optimization procedures used for search decisions of the optimum decoders. Details analyzed of efficiency decoding, which defined the transmission to OT the leading role from the classical theory in the creation of applied methods in this field of information theory.

The fifth chapter is devoted to the description of the main potentially promising methods, tools, systems and techniques that can further improve the efficiency of algorithms OT in the near future. This chapter is very important for understanding the required technological level, which must be achieved for successful further development in studies of MTD, VA and BVA, divergent coding and other methods. Only then further progress can be made in this area. Full inevitable change technological and ideological research basis of the coding theory in the field of global optimization of functionals have already taken place. Now only this approach is actually the only means of long codes optimal decoding near the Shannon's bound, which determines the current possibilities of applied methods development of the whole coding theory.

The sixth chapter briefly shows the new theory prospects, which, we expect, will be larger, than it was described by the author.

In Conclusion we formulate general results of our theoretical and applied researches.

It is assumed that our readers are familiar with the theory of probability, the basics of coding theory and main methods of the simple computing in finite fields. No unnecessary complicated relations, properties and results from the theory of finite fields will not be used in this book. This will allow everybody to focus the main effort on reading this monograph and the understanding the properties and opportunities of the studied multithreshold algorithms and realization of functional optimization various ideas, that, undoubtedly, will help the interested readers to orient well in modern problems of construction systems of noise immunity coding and with a maximum degree of assurance to choose the path to further improve the efficiency of such systems.

We must note to our readers, that due to the limited volume of the book, many sections are set out very briefly, and some of the topics that were put forward in our other publications, are abbreviated or omitted. At the same time, we tried to make all the necessary links and brief comments about the missing material. This approach has allowed us to focus on presentation of only the newest results, some of which, perhaps not yet published fully enough even in the articles on the subject. Thus, familiarity

with the questions of iterative multithreshold decoding as an optimization problem specific for the coding theory of as searching global extremum of functional we tried to make it as clear and accurate as possible.

Additional information, scientific and educational materials on OT and MTD algorithms can be found at the specialized SRI RAS website **www.mtdbest.iki.rssi.ru** and at our RSREU portal **www.mtdbest.ru**. We also remind you that for a more in-depth study OT, the MTD decoders, Viterbi algorithms in their various modifications and concatenated schemes using parity control codes (PCC) we increased in the book the number of specific links to the demo programs of these algorithms, which can also be found at our network portals. The graphic examples that characterize certain theoretical results, help to understand the optimization nature of absolutely all effective decoding algorithms for both binary and symbolic codes more quickly and deeply.

In this monograph, as an everybody can it see, there are not very many references to the works about OT and MTD algorithms, although the most meaningful ones, of course, are indicated. A huge number of references, including additional literature on this subject, are in [3-4, 36-38]. Our monograph in English published in ITU [5] in 2015 will be very useful also.

Members of our scientific school believe that the release of our monograph in this 2018, the anniversary of the great Shannon's publication [14], will serve as a further comprehensive development of coding theory in our country and in the world.

It is possible that this generalizing monograph on OT, methods for searching the global extremum of functional, and multithreshold algorithms successfully working directly near the Shannon's boundary, using simple but sometimes completely new for the coding theory paradigms, technologies and approaches, will cause certain specialists’s questions and will not be devoid of certain disadvantages. The author believes that new publications of our scientific school will smooth out these shortcomings significantly.

.