From the author

The publication in 1963 James L. Massey's "Threshold decoding" book marked a new stage in the technique development for an error correcting coding. A clear description of a very simple methods with quite satisfactory performance has defined in those years their place in various real communication systems.

The subsequent appearance in 1967 Viterbi algorithm (VA) stated the coding technique to a fundamentally new level of communication quality due to opportunity for significantly more effective coding, since the proposed algorithm provided in the Gaussian channels an optimal decoding for codes with a short length. This was the reason for attracting a large number of the theory and coding technique experts to improve just the coding effectiveness, because those years it seemed that the rapid growth of digital technologies opportunities will allow them to build a very easy concatenated and other increasingly complex coding schemes. Following this fact, the problem decoders easy implementation for a long time has been in the shadows, although the formal requirement of more simple decoding, of course, together with the task of its greater effectiveness never removed from the agenda.

Let us remind about the coding usage usefulness. The measure of its efficiency, for example, in communications technology for binary data transmission is usually determined by the coding gain (CG), the parameter which simply characterizes the value of the communication system transmitter power increase effect , when using good coding methods and, most importantly, the subsequent high reliable decoding of the received digital stream. And since this effect can reach values of 3, 5, 10 and even more times (more then 10 dB), it becomes clear the exceptional importance of the coding applications that create so large effect of power reserve in the transmitter for the communication system.

If in 1980 year a well known American scientist, the author of the classical books on the coding theory E. R. Berlekamp in one of his reviews claimed that each decibel of reduction in signal energy of the communication channel due the coding usage in those days was estimated as a million dollars, then today's scale of digital networks leades to financial increase of profit due to the coding applications in many tens of times. This very important energy resource creates the possibility for increase the speed of digital data transmission, significantly reducing the size of very expensive antennas, multiple increase of communication distance, as well as many other very important advantages of digital communication systems using noise-resistant encoding. This is particular value for the communications industry works on the creation the effective decoders. Many dozens of annual international conferences on noise-resistant encoding systems are always among the most relevant.

Therefore, the study of the majoritarian type algorithms continued, although with less intensity, because their effectiveness at first was not very high.

Despite this, in 2018 our scientific school celebrated 40 anniversary of the first candidate dissertation defense in USSR about a new iterative effective threshold

decoding algorithm. And recently we have celebrated 25 years since the doctorate thesis on the same subject, which have already been much more fully presented in the new monographs of our scientific school on Optimization Theory (OT) of noise-resistant coding [4,5]. These results have prepared the scientific community for the possibility of large-scale consolidating all results for iterative methods of error correction in a wide variety of very noisy channels with independent errors. Effect of this publication was quite strong and positive so that it allowed to our several books on multithreshold decoding (MTD) algorithms and coding reference book publisher in Russia to re-issue recently a monograph on this subject, the first edition of which was published in 2006.

Let us mark also, that for the first time in the publishing house "Science" (Nauka) in Moscow, USSR, results on multithreshold algorithms, including the Main Theorem about multithreshold decoding (MTMTD) were published else in 1981 [2].

Thus, we can assume that the Russian readers were enough informed about the state of researches on effective majority decoding, the most simple in implementations compared to other methods. The results of such advanced majority algorithms application, called multithreshold decoders (MTD) now already practically coincide with the optimum decoding (OD), i. e. they are very little different in output error probability from characteristics of total searching algorithms for the same codes even near the channel capacity. This was shown by the participants of our research school on Optimization Theory as theoretically, so and practically when modeling the work of the relevant procedures for special codes satisfying a number of very strict but clearly understandable requirements, as well as when creating coding systems at the FPGA in a number of firms. Decoders built in line with the following in the book principles have already successfully implemented in numerous communication systems.

In all cases software and hardware implementations of the MTD methods the author and developers of communication systems had got expected capabilities. Five generations of MTD decoders were created in the Institutes of the Ministry of communications in RF. It is essential that even that far years MTD characteristics were sometimes completely not available for other known error correcting algorithms with reasonable complexity of implementation.

Our book offers already well-proven working technologies for creating decoding algorithms for the four main classical channels of the theory of noise-resistant coding with the minimum possible theoretical linear complexity with the code length and at the same time (!), with an efficiency which is almost identical to the optimum decoder (OD), i.e. with a full searching, exponentially complex, error correction method in almost the entire domain of code rates R, smaller, than channel capacity C, of course, without the R values, already directly very close to the absolutely elastic and therefore fundamentally unattainable Shannon's boundary. The width of this inaccessible for MTD decoders area has been gradually reduced in recent years and, apparently, with a reasonable complexity of these algorithms, will

be slightly reduced in the near future. The issues of the current and projected level of OT efficiency will be further comprehensively addressed in the next chapters of our book.

We have long a firm belief that there are no other decoding methods comparable to our algorithms with the same complexity of implementation (the number of simple operations performed by the decoder). We offer our readers to see it for themselves.

Let us remind those basic statements, which in fact allowed us to have raised the efficiency of extremely simple threshold type algorithms up to the level of the optimal exhaustive searching procedures. They consist of only two items that provided the solution in principally new for the coding theory the problem of improving the quality of the majority decoding implementing the procedures for the searching global extremum of functionals, which have the very large number of variables, in the specific discrete spaces.

1. The majority (threshold) algorithms can be extremely effective. There are very simple MTD algorithms which have the property of strict approach to the optimal decision at all steps of the decoding until decoder changes the symbols of the received message. Under such an approach the task of decoding becomes the problem of finding the global extremum of the functional with a large number of variables in discrete spaces that thousandfold pushing the research horizons of development and application for principles the optimal errors correction.

2. The effect of errors propagation (EP) in the threshold decoders is really strongly limits the ability of the majority decoding procedures. But this effect is completely controlled. Its correct interpretation helps to form requirements and criteria by which other optimization based procedures can be constructed for a searching codes with a very low level of error propagation on the output of the decoders corresponding to them. It allows extremely improve the efficiency of iterative threshold type procedures.

The first property turns out to be essentially completely unexpected. But, indeed, after a little, but principled modification of the usual threshold algorithm that turns it into multithreshold decoder (MTD), the new algorithm really becomes having unique property of the approaching to the optimal (with an exhaustive searching!) decision if the very simple conditions have been met. And since in this case the algorithm for all changes continuously calculates strictly the distance between the received vector and the current solution-hypothesis about the transmitted vector, then it actually implements the global total search procedure (specifically: the minimum of this distance) when they have just linear growth complexity of the algorithm with the increase the code length.

It seems very plausible, that no other known methods of error correction have similar MTD properties now. A new understanding of decoding as a search the global extremum of fuctionals connects to coding theory absolutely grandiose number of theories, methods and styles created by different global optimization theories for the

fastest possible searching this extremum in the very difficult conditions of very small signal/noise ratios of digital communication channels.

And the second of the above statements deserve a long time discussion and serious justification, as it is done in one of the chapters of this book. A successful solutions for this complex problem of EP indeed allowed to create codes that are particularly effective when they are used just in MTD.

We hope that for many natural questions about complexity, efficiency and adaptability of coding and multithreshold decoding the readers of this book will get enough meaningful answers. In case of interest they can surely continue a very promising for all communication systems investigations in the MTD decoding procedures, which have already found their place in a number of developments.

As it will be shown later, when creating new algorithms and the codes corresponding to them the main task of the researchers becomes the most accurate and optimized for a lot of parameters simultaneous design the decoder and code used in it. In other words, the simplicity of the MTD implementation is achieved by more time-consuming and carefully organized stages of design code and specific algorithms for its decoding. In this case the problem of implementation complexity of the algorithm purposefully transforms in such a way that the technological tasks of more efficient decoder construction was solved precisely by those components of complexity, the increase of which is the most available or even usefully. For example, in the most cases of minimization the amount of decoding operations when MTD has comparable efficiency at $2 \div 3$ decimal orders less than for other algorithms, precisely due to the significant memory size of the decoder that uses very long codes. It is clear that this is sometimes absolutely necessary in high-speed communication systems with a high noise level. But with all the complications of the design procedures of MTD algorithms, the decoders themselves created by these new methods always remain the simplest majoritarian methods with the characteristics of practically optimal error correction algorithms, even for very small signal/noise ratios of the communication channel.

It is useful to point that when closed to the channel capacity algorithms MTD maintain a real modest complexity, quite affordable for modern technologies. But other techniques with the same complexity (number of calculations) in this realm of noise are usually generally unworkable.

More systematic than before, the presentation of methods and paradigms OT, as well as new results obtained on their basis, allows, as the author hopes, to get enough in-depth understanding the OT status and usefulness of its applications. The main scientific result of OT presented in this book, is a long-awaited achievement by the MTD algorithms that form the OT basis, the realm in immediate vicinity of the Shannon's bound in the most of channel types traditional for coding theory.

The book describes the whole a number of different methods and approaches to achieve this main target in the information theory. All of them really allowed us to

solve ideologically the most difficult problem at a quite acceptable level of technological complexity of the proposed algorithms.

The importance of high quality codes according to the criteria of the small error propagation (EP) must be emphasized also. They are very important for MTD algorithms. These codes must be very long. Solved in this monograph the problem of the maximal approximation to the bound of Shannon got its practical realization in the real and very simple schemes of MTD decoding for all types of channels with independent errors just due to the fact that such codes managed to build. The distance of MTD algorithms workrealm from the Shannon's bound in Gaussian channels is equal to only ~1 dB. A very small difference between the channel C and code rate R in erasure and non-binary channels shows totally a complete success OT in solving the problem of effective operation of the simplest algorithms near this absolutely elastic and unattainable boundary.

It is especially useful that simple MTD decoder even for extremely large noise saves ability to achieve with its own linear with the code length complexity the optimal exhaustive searching decoder decisions, providing maximal possible reliability of its result. It allows to create hardware MTD versions of these algorithms with the theoretical maximum speed also. However, decision delays making due to the large number of iterations in the convolutional MTD and significant code lengths are for little EP quite large, reaching in some cases the values of several Megabits. This is inevitable when there is a large noise level near the channel capacity and is quite acceptable and even necessary for many satellite and optical communication lines.

After solving by OT methods the fundamental problem of simple decoding set by the Shannon, the next stage of OT development, we believe, will be a creation of shorter delays in MTD procedures within the framework of OT, which will be very useful at smaller decision-making delays, including block codes, although for these codes in our book already referred many good implementations of the MTD procedures and our patented block Viterbi algorithm. One of such methods that already provide a significant reduction in delay of MTD decisions are simple and very easy concatenation work of the MTD with a very short external codes. There are a lot of corresponding publications already. These solutions are already described in the third chapter of our monograph. And the other promising approach to this task is to replace the simplest threshold function in the MTD by slightly more complicated procedures that will be also extremely useful for diminishment the number of decoding iterations in all main channel types. The block decoder realizing convergence may be useful also.

Perhaps a more complicated decision rules for iterative decoders within the OT will lower the requirements for the used codes, that will reduce decision delay due to a small (!) increase the complexity of calculation, which will be especially useful for block codes also. This task, possibly may be solved by researchers and developers in

OT and MTD in the years that follow. Our block variant of Viterbi algorithm can help to solve many problems in block codes too.

The attentive reader will certainly note that many of the properties and the possibilities of MTD algorithm presented in this book and others methods relating to the OT, as in the previously published monographs, repeatedly examined and commented in various sections of the book from different perspectives. The author is agree that it is so. And in this monograph (as in the previous ones) it is also made for the sole purpose of the most full, comprehensive and at the same time, the most understandable evidence or explanation not a very common features of multithreshold decoding, various modifications of the Viterbi algorithm (VA), Main Theorem of multithreshold decoding (MTMTD) and many other new OT paradigms. The way of presenting the material is dictated by the fact that, although all the key results are obtained by rather simple methods, many of them are still not previously used in publications on theory and technology in coding and are completely new for this branch of science. It requires a very careful and gradual presentation of many cases quite not simple and sometimes even unexpected results, properties and characteristics of our algorithms. This variety of commentary and forms of presentation, as we see it, facilitate the reader's task of understanding presented in the book results for learning which sometimes still require the considerable efforts and time.

Additional information on MTD and OT is proposed in [5,40].

We hope very much that a large support for readers will give materials of our bilingual and, we believe, the world's largest network portals on coding theories, MTD algorithms and OT: **www.mtdbest.ru** of RSREU and **www.mtdbest.iki.rssi.ru** of SRI RAS. They have presented more than 600 scientific-reference and research-methodical blocks of data. Now they have not only demo programs for error-correcting algorithms of the most known in the world, but also our new software platforms for our different decoders. Our readers are kindly welcome to copy to their computers these new convenient software, and immediately start study and research MTD decoders. They will work in the majority algorithms field with changing codes and their studying parameters, as well as the characteristics of the decoding algorithms and channels within very wide limits, in accordance with all the traditions of full-fledged scientific work. This advantage of approach to the preliminary studies our foreign colleagues already saw, who praised the new features provided by our software platforms for a closer acquaintance with OT and its paradigms, algorithms and technology. Usage flexible software platforms allows them to expand significantly the scope parameters that can be controlled in the experiments with MTD decoding, including the codes that can be changed at the request of the experimenter. The usefulness of this type of joint work is well confirmed by the successful start of the foreign patenting stage of MTD algorithms. This activity will also develop.

The experience of publishing our Handbook on coding [1] shows that it was also useful for many readers to study various methods of error correction, including MTD decoders.

The author considers, finally, it is useful to emphasize that all the assumptions of the study, theoretical results, the practical consequences and conclusions arising after the results of the presenting study continues to be extremely simple. They are associated only with the most common notions of the probabilities theory and applied issues of the error-correcting coding and do not require knowledge in special parts of other difficult disciplines. This is the opportunity to take a look on the potential of codes and multithreshold procedures, based only on the simplest theoretical considerations and common sense. It creates the conditions for very rapid learning students and experts new opportunities of coding based on MTD algorithms, very simple methods to achieve optimal solutions based on theories and methods of searching global extremum of functionals.

The author would like to thank many assistants, members of our scientific school, enthusiasts and highly skilled professionals who have been helping him in his researches for many years and in applying the results obtained in specific systems and projects. They gave the opportunity to write this book.

The author is also grateful to his colleague doctor of technical sciences (Dr.Sci.Tech.), Professor G. V. Ovechkin, which directs many studies in the field MTD and OT.

Key support for the work on MTD has been provided by MIPT, the Scientific Council on Cybernetics of the USSR Academy of Sciences (AS), the Nanotechnology and Information Technologies Department of the Russian Academy of Sciences (RAS), the Scientific Research Institute (NII) NII-QUANT, the Radio Scientific Research Institute (NIIR) , the Voronezh Scientific Research Institute of Communications (VNII-Svyazi), the Bonch-Bruevich Electrotechnical Institute of Communications (LEIC) in Leningrad/St Petersburg, Space Research Institute of the RAS in Moscow and the Ryazan State Radio Engineering University (RSREU).

Probably, the fifty years of MTD study could not be presented in its up-to-date form without support of RAS academicians A.I. Berg, V.K. Levin, N.A. Kuznetsov, L.M. Zeleniy, corresponding member RAS U. B. Zubarev, professors E.M. Gabidulin, S.I. Samoilenko, Yu.G. Dadaev, V.I. Korzhik, A.N. Pilkin, V.V. Vityazev and doctors of technical sciences S.V. Averin and R.R. Nazirov, who praised the submitted research materials, participated in the analysis of MTD algorithms and greatly contributed to their acknowledgement by scientific and technological community in Russia.

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It is obvious to us that we should translate and edit our book especially carefully. This is determined by the fact that the problems of our theory and coding techniques have been available to Russian readers for a long time. But the specifics of

our many assertions and comments in English, which set out a completely new ideology of decoding, can create some inconveniences of their understanding. However we believe that the interest of this book readers to OT and algorithms MTD will be the main feeling and would help them to see the new theory essence. In this case we will consider that imperfections of our work in OT transformation from one language into another did not prevent correct understanding the results. We invite you to learn new "quantum mechanics" for XXI age in the information theory together with us.

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To obtain a visual impression about the MTD effectiveness our readers are offered a small demonstration animated cartoon. This demo program designed to run at IBM PC a Win-compatible computer in which some of the most important features of the MTD procedures are illustrated: multithreshold decoding in the classic binary symmetric channel (BSC) under high noise conditions. Based on the publishing experience of our previous books on the OT and MTD subject such a small preliminary psychological preparation with proposed software demo will create the necessary emotional and epistemological preconditions for fruitful follow-up working with this book and with our new algorithms. Instructions for work with the demonstration program and the cartoon itself can be copied from a specialized portal SRI RAS **www.mtdbest.iki.rssi.ru** at the main description page "**About method**" or at the similar portal of RSREU **www.mtdbest.ru (**giperreference **guide** and **demo program)**. Cartoon-film clearly shows the process of strict reduction of the current decisions of block MTD distance to the message received from the channel. It illustrates the work of simple, but a very powerful process of searching global extremum (concrete: minimum distance) of functional on the MTD basis very well.

You can also find there a variety of additional information on MTD algorithms, educational and methodological materials, a large number of articles, books and OT presentations, its paradigms and technologies, as well as on the block Viterbi algorithm (BVA), divergent decoding, decoders with direct metric control and other issues.

A number of problems related to general ideas of coding and specific capabilities of MTD algorithms, are considered on our websites as well in sections of answers to questions which allow you to assess the capabilities of coding systems based on MTD algorithms and general OT principles more accurately.

There is also correspondence with the editors of scientific journals. It also helps to understand the conditions for the development of researches in the OT field. Some of these issues are submitted in Russian only.

Significant support in the study of decoding methods based on MTD algorithms, our readers can get in three very useful laboratory works that could be copied from educational pages ("Education") of our network resources specified

above (hyperlink [72]), and offer to radio-technical departments of Universities for students and profession-in-house retraining for specialists in the field of telecommunications. (The first laboratory work you can use at once. The next two works should be taken from Russian part of site and translated. You may be sure that it would be very useful for good subject understanding .)

For more information on the multithreshold decoders and other useful methods of error correction, please, look for our reference book [1] and the monographs [2-4]. Our monograph [5], published in Geneva, would also contribute to a rapid and complete understanding the fruitful ideas of majority decoding near the Shannon's bound. Many of the MTD and OT problems in [5] are described more specifically, for example, software ultra-fast version of the algorithm MTD and methods for assessing the errors propagation (EP) for different binary codes.

Your opinion concerning this book and suggestions for its improvement, please, send at the address: Russia, 117997, Moscow, Profsoyuznaya str., 84/32, SRI RAS, Department 71, V.V. Zolotarev, or by e-mail: **zolotasd@yandex.ru**.

Sincerely yours,

Author