# Question 27

How else can you tell us the work of the MTD in another way? The descriptions in the books are quite formal. For some readers, this is difficult to some extent.

Can we look at the Zolotarev’s decoders in a more imaginative way?

# Answer to question 27

How else can we look at the MTD work?

One of our students looked at the scheme of the Zolotarev’s decoders in our comics about OT and MTD and in a minute drew this picture. And when we asked him what is this, he explained to us that this is how he sees the MTD process.

**The convergence of the procedures of global search for MTD decoders**

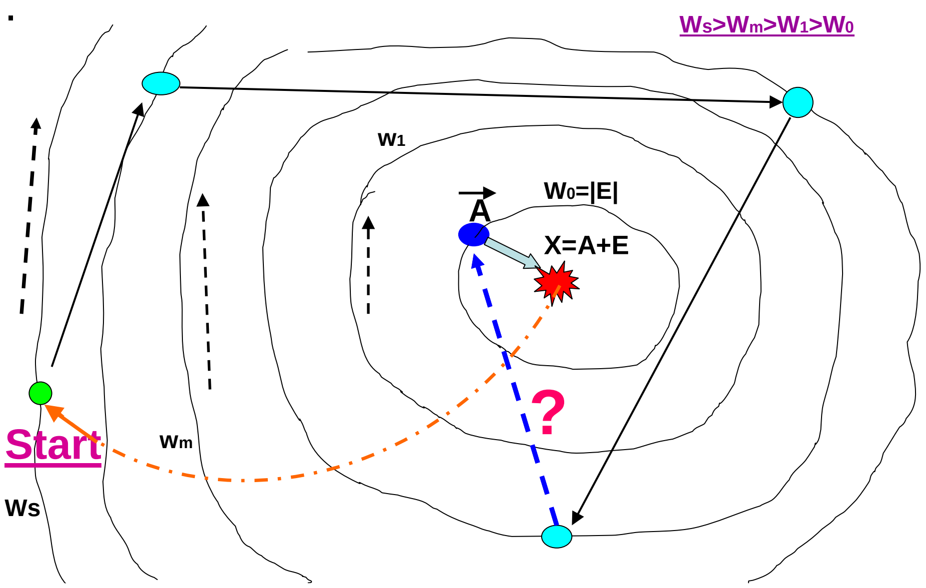



Fig.I

Since the code under discussion is linear, let's assume that there is a high probability of distortion in the transmission of code characters and a null message is sent. The choice of the transmitted message, which is a code word, does not affect the probability of an errors in the decoder decisions. During transmission, the noise vector E was added to the code vector A, whose weight is equal to |E|. In our case, the weight is the number of units in the binary vector. The decoder will receive the vector X=A+E. OD must find such a code word that it would be the closest to the vector X among all possible ones. Since A is a zero vector, the weight of X is the same as that of E : |X|=|E|. And let the message A be the closest to X, i.e. A is the solution of OD.

We immediately asked the student how he would tell us about his image if the noise vector has such a large weight that A is not the decision for OD. And he almost immediately pleased us with a clear answer that in this case absolutely nothing will change at all, since he talks about the process of convergence of MTD decisions to the solution of OD, and not to the correct solution. So in order not to interfere with the understanding of the image, our student immediately suggested that the vector E is not so "heavy" and the sent message A is exactly the OD decision.

And then it turned out to be even more interesting. As we know, in MTD, as in many other decoders, the simplest procedure for calculating the syndrome vector for the received message is performed first (see about this operation in the books).

And then we get certain initial version of the MTD decision, from which the process of finding a solution to the OD begins. This option is shown by the blue “Start” dot at the edge of the drawing, additionally marked with the Ws symbol . This sign indicates the weight of the difference between the start point and the received X vector. And this point lies on a certain circular line, on which are all decisions that have the weight of the difference with X equal to Ws. The weight of the Ws is always really much more than |E|, Ws>>|E|. On the inner circles lie other decisions, the distances Wm to which from the vector X decrease as they approach to X. The total number of decoder decisions lying on all different circles increases exponentially with the code length n.

In this case, the decoding process becomes such that, according to Fig.**I** of the block MTD, the data in it moves synchronously and cyclically. The threshold elements gradually changes and improves the controlled information symbols. And in the diagram under discussion, during error correction, the decoder looks at the current Wm weight circle (but first, of course, on the Ws weight circle !) for the ability to move from it to one of the inner circles, each of which is a geometric place of decisions that are at some equal distance Wn from X, and Wm>Wn. However, the threshold elements in the MTD are such ones that the transition to the inner circle is possible only when the new decision at this inner circle of lower weight is different from the previous decision in exactly one information symbol. And this condition is the greatest problem of majoritarian (threshold) algorithms, which was formulated and then successfully completely solved by the theory of errors propagation (EP). But if it is not solved, then usually the decoder will run around a circle of some weight Wn, Wn>$W_0$ after some moment on. The possibility of such a failure is shown by the last dotted arrow to the decision A, which is emphasized with a red question mark the lack of guarantee that the algorithm will achieve the final correct decision A at the end of the decoding process . But if you use all the features of the EP and build only the correct codes according to the criteria of the EP, then even with a lot of noise, the difficult search on the circles of the student's diagram will almost always continue until the decoder gets to the decision A, which is closest to the accepted vector X, as we agreed before.

Our student also said that the current condition for the uniqueness of the changed symbol in the next MTD decision after each message correction event is very strict, but it is not necessary at all. He suggested searching for other crucial rules for Zolotarev's decoders in order to change many symbols or even in some other way, which can improve the decoding characteristics at high noise levels. It is quite obvious that the main condition here – preserving the linear complexity relative to the code length n, - can be fulfilled.

But the search a specific type of new decision function is a very difficult task. It may soon be solved by our student or other participants of the OT scientific school. This will be the next step in the further development of the error propagation (EP) theory in Zolotarev's decoders. I wonder who will solve it. Yes, who will do it? A student, we, or maybe **YOU**?

We are agree.

## Successes for you!

### Question 28

Why is the MTD theory so complex? And how to evaluate as simple as possible the characteristics of Zolotarev's algorithms?

## Answer to question 28

## About the OT perfection and its compactness.

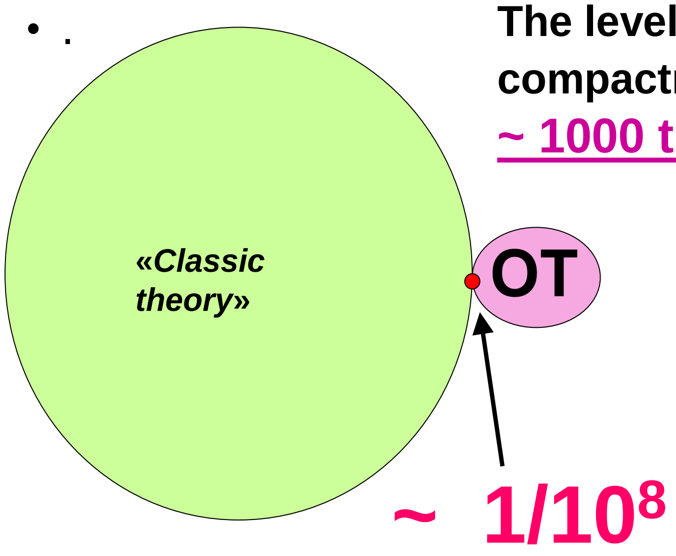
A very important question is the place of OT and its relation to the classical coding theory. A very conventional high-quality picture illustrating these relationships is the slide below, which nevertheless provides clear answers to these questions.

The main answer: on the contrary, it is extremely simple. This is obvious because the basis of OT is Zolotarev multi - threshold decoders, which are majority decoders, i.e. the simplest algorithms known for 50 years. Their difference from the decoders of J. Massey is fundamentally from the point of view of theory, but in essence these are the same decoders of the great American scientist, changed very slightly, but so that they began to approach the solution of the optimal , i.e. the best possible minimum error probability of decoding (OD) at every step.

## The sizes of «*classic*» and OT

The level of OT compactness

~ 1000 times!!!




An analysis of all the OT monographs published by our scientific school in this Millennium shows that they are all system-philosophical treatises on information theory with extremely limited usage of complex mathematics. Some exceptions are those sections of OT that relate to the basics of error propagation theory (EP) and its applications to specific codes. It is quite clear that such a new important and very original section of coding theory, which no scientific groups in Russia and abroad have been able to discover, research and understand over the past 50 years, could not be very simple. And yet, after a complete analysis and description of such a complex phenomenon as EP, the conclusions and technologies that followed from its understanding were also clear and natural. EP has shown that it is necessary to construct codes with the lowest possible dependency of checks relative to decoded characters among themselves. In other words, the codes should be such that for any pair of decoded characters, the number of common errors in checks should be the minimum possible. And this can already be achieved based on clear computational processes. This conclusion, which cannot be made without EP theory, led to simple algorithms for creating such codes, which also fit into the concept of simple methods with clear and understandable final criteria for the quality of codes.

The total number of new mathematical relations in the characteristics of OT methods turned out to be quite small and is now limited to just a few dozen new simple formulas. And from the previous applied coding theory, only a few simple transformations are used in OT, which allow us to calculate the probability of error in the first character of the used binary block or convolutional code $P_1(e)$ for a binary symmetric channel (BSC). The slide shows a chain of simple calculations based on generating probability functions (GPF), which allows us to calculate this parameter $P_1(e)$ of the majority decoder, which is convenient for all preliminary estimates.

## The overall result is the only one

The probability of the threshold decoder error in the first character $P_1(e)$ of the code.

Since the check is incorrect with probability

$p_J = 0.5[1–(1–2p_0) J]$ .

Then through the PFV of the form

$$A(x) = (p_0 x + q_0)(p_J x + q_J)^J = \sum_{m=0}^{d} a_m x^m$$

getting the probability of an error in the first character

$$P_1(e) = \sum_{m=(d+1)/2}^{d} a_m$$

**AND THAT'S IT!**

All this, however, allows us to calculate all the necessary characteristics of decoding by OT methods in all classical channels considered in the coding theory for any algorithms of our new "quantum mechanics" in information theory.

The described situation is presented on the slide, which shows the almost completely independent former "classical" and new optimization part of the applied coding theory. Connecting them is a small red spot, with which we have designated several expressions that make up the proposed J.Massey method for calculation the probability $P_1(e)$ is all that unites them. OT - is a completely rewritten applied coding theory. And the ratios shown on the slide for the dimensions of these two theories do correspond roughly to three decimal orders of the informational volume. The conditional share of formulas for $P_1(e)$ is really insignificant and the specified part of them is actually only their qualitative assessment. Well, finally, we note that the entire OT is a very clear and completely understandable theory. So – learn it please!

OT – it's very simple! But the results are the best, optimal!

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But had OT really solved the Great problem of Shannon, stated 70 years ago? Positive answer this question was formed in all our monographs of this millennium. Indeed in these books many results were submitted which had shown that for block and convolutional, binary and non-binary, low-, middle rate and high rate, erroneous and erasure, basic and concatenated (and others!) codes we now can construct MTD and BVA decoders with extremely high characteristics near Shannon’s bound. We had checked and analyzed ~ 100 such very different clusters. Our scaled work here clear proved it: - Shannon’s problem is solved really! We and you may turn to solve next more difficult problems of information theory.

Well, Good Luck to you and us!