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An Effective Algorithm of Noiseproof Coding for Digital Communication Systems

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Information transmission through communicational channels becomes considerably more difficult because of the disturbances and noises in the channel. An effective mean for increasing of truthworseness of transmitted information is a noiseproof coding. There is a comparison of basic algorithms of noiseproof codes decoding, based on the materials of scientific-technical conferences and articles about the coding theory in this survey.

Criteria of coding efficiency. During 12 years after the moment of publication of the review article [1], the coding theory has moved far ahead. More powerful codes and methods of their decoding have appeared, providing a work of communication systems near the theoretical channel capacity. The most common index of qualitative estimation of decoding methods is a code gain (CG), showing a lowering of energy required for transmission of one data bit (with a certain chosen bit error rate $P_b(e)$) in the case of usage of some coding and decoding algorithm, in comparison with the case when there is no any coding.

Foreign specialists more than 20 years ago valuated every 1 dB CG in million dollars [2]. Now the price of CG has grown up even more, due to the possibility to minimize sizes of very expensive antennas or to increase strongly communication distance, increase transmission speed or to lower indispensable power of transmitter, to prove others important properties of modern communication systems. Note that the price of modern communication networks is growing up all the time very fast and absolutely can not be compared with expenses that were possible some years ago. Mutual noises for acceptance between different systems considerably increases, more and more ecological limits on power of transmitters appears, grows necessity in quick increase of data exchange speed between these networks, and also in a considerable growth of it's truthworseness.

Further in conditions of economic globalization and explosive (very fast) society informatization, these trends will become even stronger. So we can make the conclusion that increasing in a few decimal powers economic (and ecological, and generally – systematical) profits of CG based on the noiseproof coding methods becomes extremely actual and requires using of theoretical achievements in the sphere of noiseproof coding for computer communication systems creating.

Decoder's characteristics. Fig.1 shows dependency of a bit error rate $P_b(e)$ at the output of the decoder on signal-to-noise ratio E_b/N_0 in binary symmetrical channel without memory (BSC). The errors arise in a hard-decision modem with binary phase modulation in the channel with additive white gaussian noise, when they do not use

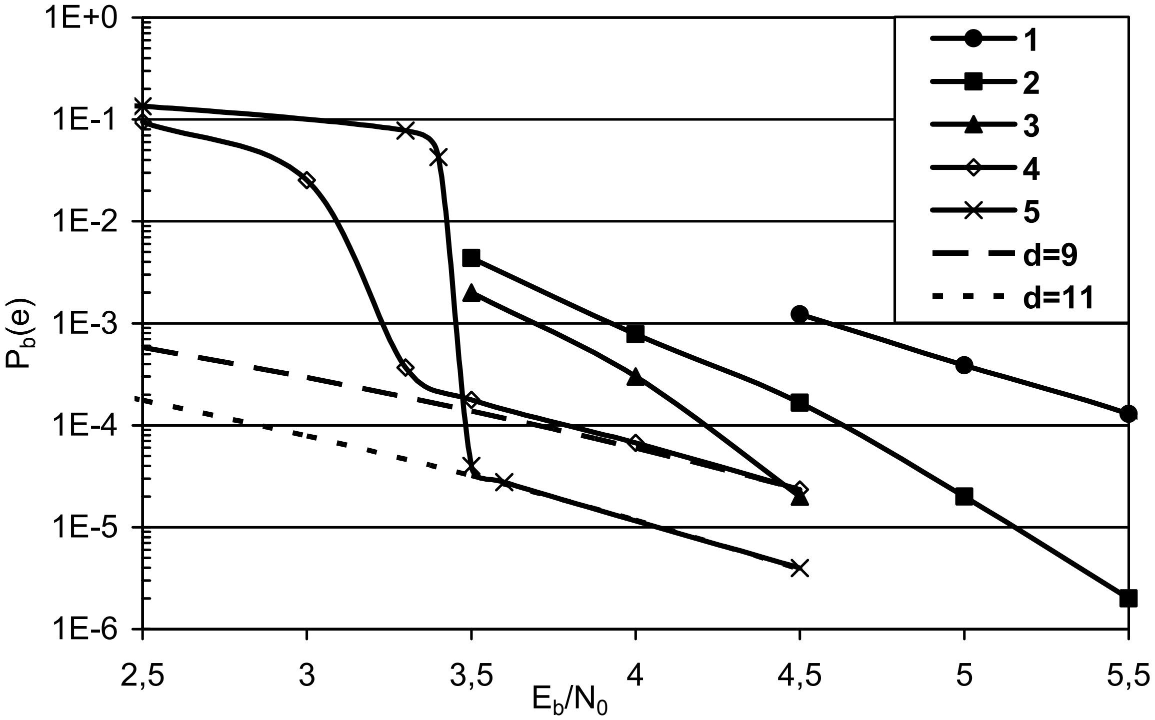

Fig.1

in decoder estimations of truthworseness of accepted symbols. Such a channel's model quite exactly describes real satellite and some other types of channels.

Curve 1, 2, 3 correspond to a widely used Viterbi algorithm (VA) [3, 4] for a code rate R=1/2 and length of coding register k=7, 11 and 15. This method – is an optimal one but its complexity growth exponentially with growth of k and that is why decoder with k=9 do not use practically. As a result the necessity arises the searching problem of more simple (from the practical realization point of view) decoders, providing at the same time decoding, close to the optimal one.

Multithreshold algorithm. One of the simplest error correcting algorithms – is the threshold decoder (TD) of Massey [5], taking solution about the value of every decoded symbol on the basis of simple «voting» of checks. The scheme of the threshold decoder of convolutional self-orthogonal code (SOC) with code distance d=5, code rate R=1/2 and the constraint length of code n_A=14 is shown at Fig.2. It consists of 2 binary registers, some semi-summarizers (summers on mod2) and threshold element T, that just sums binary checks of the decoded symbol and compares it with a threshold, changing this symbol and all checks, relating with it, if the sum of checks exceeded a certain threshold value. But this method has a pure correcting ability, so it is rarely used in data transmission systems.

The better characteristics have Multithreshold decoders (MTD) [6], representing the modification of TD, introduced above. MTDs in the process of control of received code sequences correct repeatedly informational symbols of message, arrived from the channel. These decoders have the most important and strictly proved property – convergence to the optimal decoder's solution (OD), keeping linear on the code length growth of realization complexity.

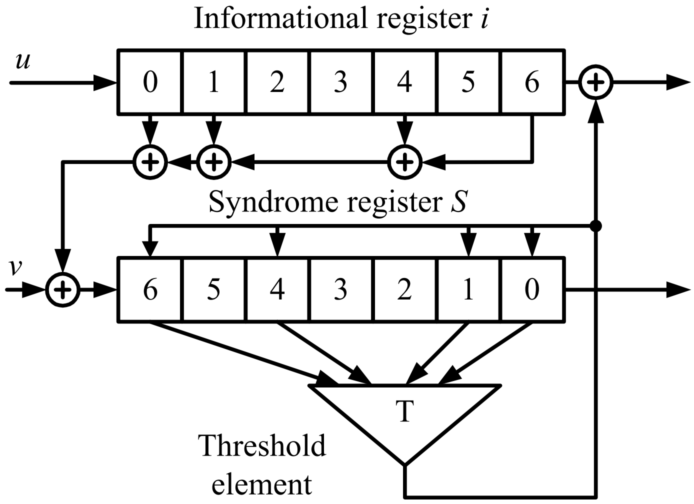

Fig.2

Scheme of MTD for convolutional SOC with d=5, n_A=14 and two decoding iterations is shown on Fig.3. At bigger number of iterations that is usually necessary for work in the area of high channel's noise, further iterations are absolutely analogous to the second one. As it is obvious from the scheme, every iteration of MTD differs from the usual TD only in the presence of «differential» register, where informational symbols changes can be matched by threshold element (TE). It is important, that data from differential register are used then by another TE at the following decoder iteration. At each MTD iteration at decoding of informational symbol i_k on TE – the only active decoder's element (if hard-decision modem in BSC is used) – the following operations are done:

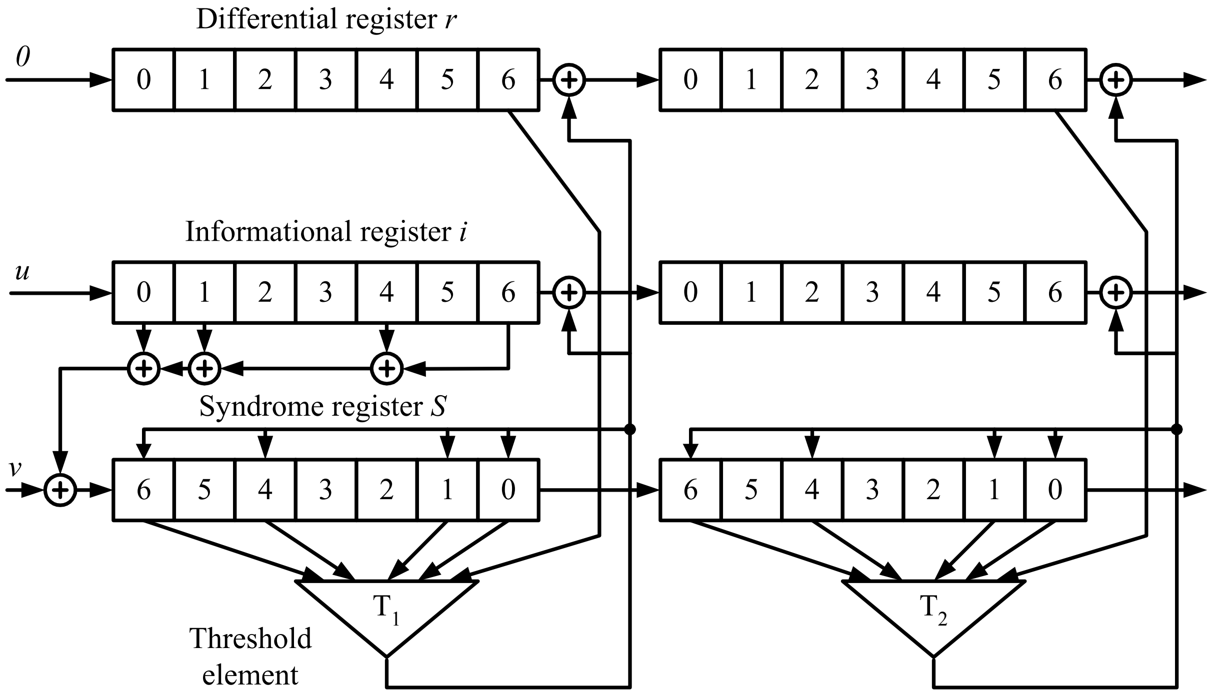

Fig.3

1. Sum of all checks is calculated (each of them for case of BSK is 0 or 1), including corresponding symbol r_j from differential register

$$L_i = \sum_{m=1}^{J} S_{g_m} + r_i;$$

2. If sum of checks L_i greater, than T, where $T=(d-1)/2$ – is meaning of threshold element, the informational symbol i_k, all checks connected with it and symbol r_i are inverted.

3. Transition to the next symbol decoding (p.1)

Note that when soft-decision modem is used, i.e. when truthworseness of each binary symbol is estimated in gaussian channel, in MTD there are the same operations, but checks are summing with scales, defining truthworseness of estimations of bits received from channel. Threshold T for decoded symbols will be changeable, not fixed as in case of BSC.

Analyze of MTD characteristics. Curve line 4 at Fig.1 corresponds to the MTD work with convolutional SOC for R=1/2, and minimal code distance d=9, constraint length of code n_A=4168 bits and I=15 iteration of decoding [6]. Abilities of MTD with R=1/2, d=11 and I=20 are shown at Fig.1 by curved line 5. The dotted line on the figure shows probability of erroneous decoding of the code with d=9 and d=11 for optimal decoder. As it follows from figures, MTD (for a rather long codes) occurs even better than deliberately non-realizable optimal AB with k=15 and more. Let's mark that with growth of code length and increasing of iteration number the characteristics of MTD converge to characteristics of optimal decoder faster.

Fig.4 shows the characteristics of the same decoding algorithms at usage of soft-decision modem, estimating the truthworseness of received binary symbols with

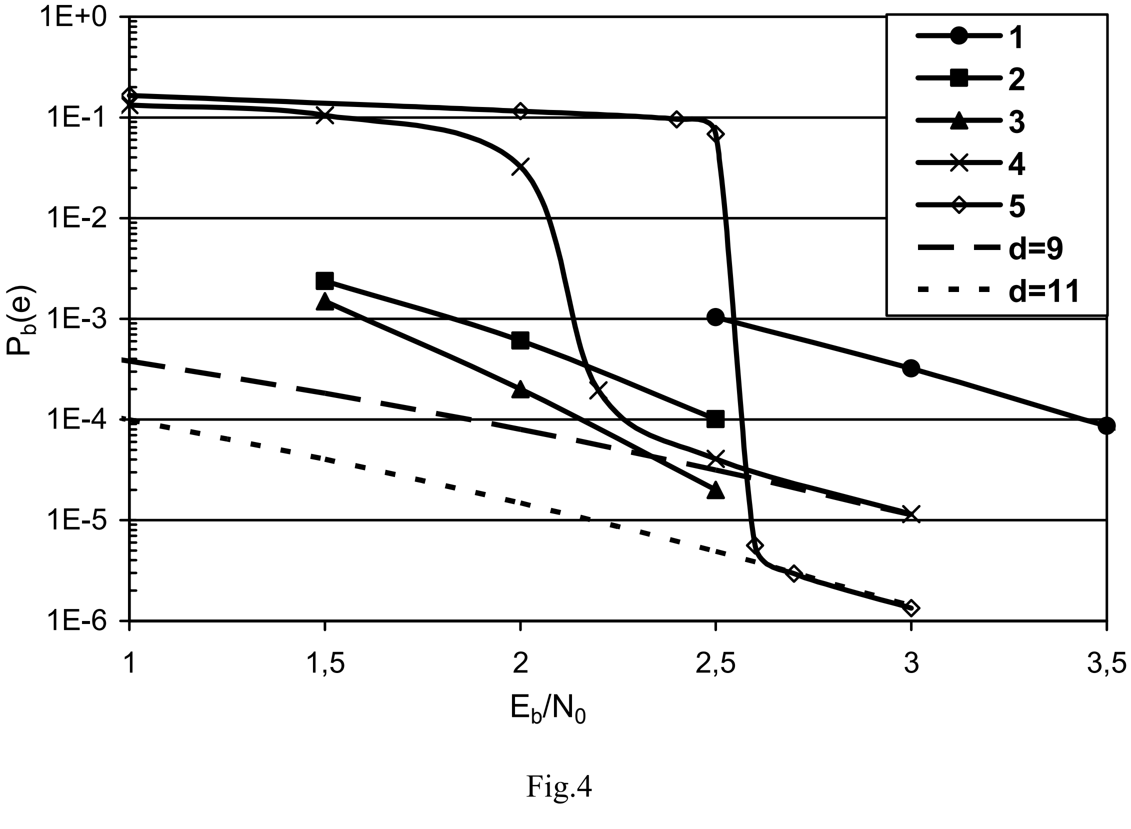

Fig.4

binary flow quantizeation at Q=16 levels for codes, mentioned above. Comparison of Fig.1 and 4 shows that the move to the soft-decision modem at usage AB allows to get additional CG near 2 dB, and at usage of MTD – 1,4…1,7 dB. But even in this case MTD, keeping the simplicity of the usual TD considerably exceeds AB in efficiency. Basic correlations between these algorithms can be at another code speeds.

Facts, discussed above, show that available for realization optimal AB yield to MTD, where could be easily decoded long codes, in wide diapason of code speeds at usage of hard- or soft-decision modems. And MTD does only the simplest operations of comparison and addition of very little integer numbers that is why it is a very simple at all variants of soft and hard realization.

Considerable improve of characteristics of decoding provide concatenated coding methods [7] at low complexity of realization comparing with non-concatenated. And then initial data are coding with a help of two and more code, which then would be decoded by methods corresponding with these codes.

Efficiency of turbo codes. Let us consider turbo codes, founded in 1993 [8–10] – codes with parallel concatenation. They may be created with the help of 2 and more systematical coders, connected between each others by interleaves. This name «turbo» describes properties of iterative algorithm used in decoding: information from the exit of one iteration of decoding is the entrance for the next iteration. In decoders of composed codes usually optimal decoding algorithms with maximum of a posteriori probability (MAP – algorithm) or its easier variant – max-log-MAP (MLM) are used.

On Fig.5 characteristics of turbo codes are shown: curved line 1 – R=1/2, K=65536, MAP; curved 2 – R=1/2, K=1000, MLM. Here K – is a length of informational code block. There were done up to 20 iterations of decoding in the case of submitted diagrams. Codes show very good characteristics for a high noise level of communication channel. With their help the probability $P_b(e)=10^{-5}$ can be achieved when E_b/N_0=0,7 dB, that is just at 0,5 dB higher than theoretically achievable limit. However in case of lowering of noise level there is an error floor effect, due to low free distance of these codes.

The complexity of decoders for turbo codes is very high. For example, turbo code, based on MAP-decoder, produces 12N FLOP/IB/Iter, and on MLM – 30N AEO/IB/Iter operations. Here N – is a number of possible coder's states, FLOP/OB/Iter – the quantity of operations with real numbers (including composing and multiplication) per one informational bit on every iteration, and AEO/IB/Iter – quantity of additions, subtractions and comparisons of integers.

The other kind of concatenated codes are sequential concatenated codes [11], for which the iterative decoding scheme is used too. Their abilities are also shown on Fig.5: curved 4 – R=1/2, K=1000, MLM; curved 5 – R=1/2, K=1000, MAP. These codes have a higher code distance and so show better characteristics at middle and low noise level.

Woven codes (12–15) are also sequential concatenated codes. In coder of this code a row of outer coders and inner one seems to be woven indeed. On Fig.5 are shown abilities of wicker codes: curved 6 – R=1/2, K=1000, MLM; curved 7 – R=1/2,

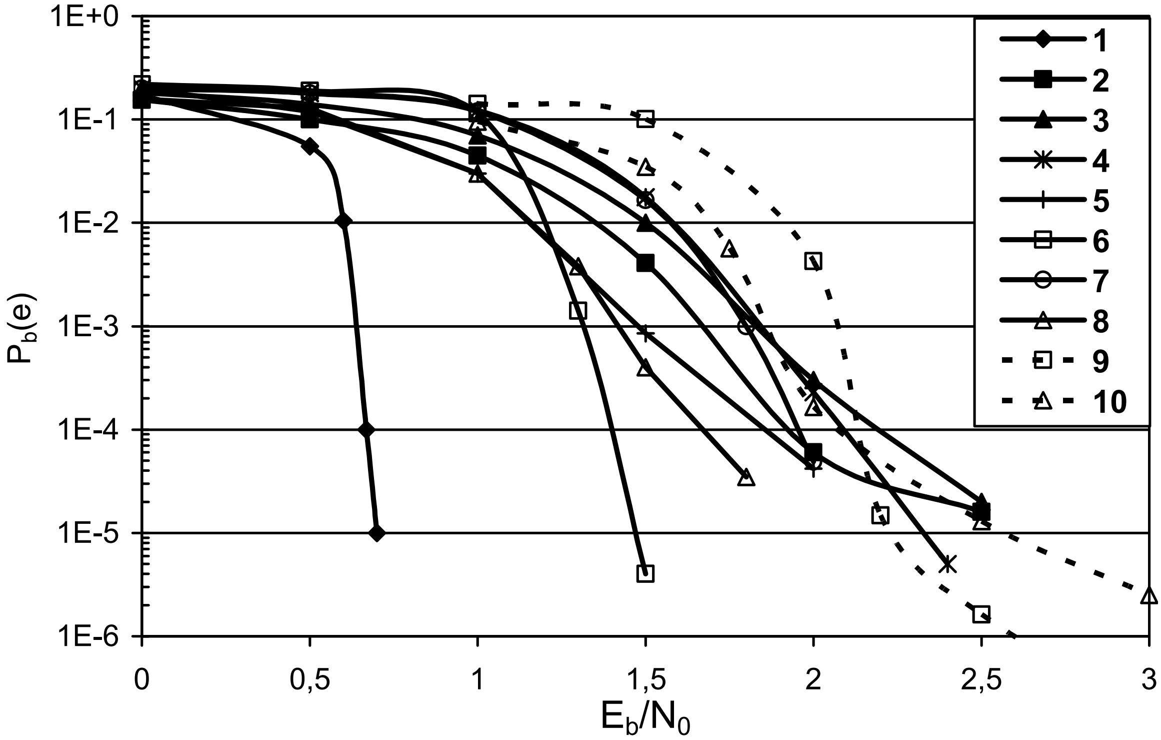

Fig.5

K=1000, MLM, curved 8 – R=1/2, K=1000, MAP. Codes are worse than turbo in case of higher level of noise and better at middle and low noise level.

Fig.5 shows that in case of work in temperate noises MTD that is characterized with a high throughput and real simplicity (MTD with d=9 does near 12 AEO/IB/iter operations), that is common for all majority algorithms, it does not yield to concatenated schemes considered above. Besides with such a noise there are errors in the output of MTD basically single [6], that allows to apply it successfully in very simple concatenated codes even without using interleaving.

Curved lines 9 and 10 on Fig.5 show abilities of concatenated schemes on the basis of MTD and parity check codes (PCC) (with d=2): 9 – MTD with R=1/2, d=9, n_A=3052 and PCC with n=50. Using of this concatenated scheme allows to reduce error probability of decoding in 10–100 times comparing with the simple MTD.

Let's underline that number of operations in MTD (anyway is a very small) could be reduced in some cases even approximately in 4 times without loosing an efficiency of decoding, that considerably increases the speed of work of this algorithm.

Comparison of decoding algorithms. Finally note the comparison of algorithms with consideration of CG providing by them. Fig.6 shows the energetic efficiency of algorithms of decoding in case of work with hard-decision modem: curved 1 – AB with K=7, R=1/2, curved 2 – MTD with R=1/2, d=9, curved 3 – MTD with R=1/2, d=11; curved 4 – concatenated: PCC with n=50 and MTD with R=1/2, d=9. Fig.7 shows CG of decoding algorithms in work with soft-decision modem: curved 1 – AB with K=7, R=1/2; curved 2 – turbo with R=1/2, K=65536, MAP; curved 3 – turbo with R=1/2, K=1000, MLM; curved 4 – MTD with R=1/2, d=9. Concatenated schemes on the basis of MTD with d=9 and d=7 are shown as curved line 5 and 6.

This figures show that MTD and concatenated schemes on its basis with small $P_b(e)$ do yield a bit to turbo codes.

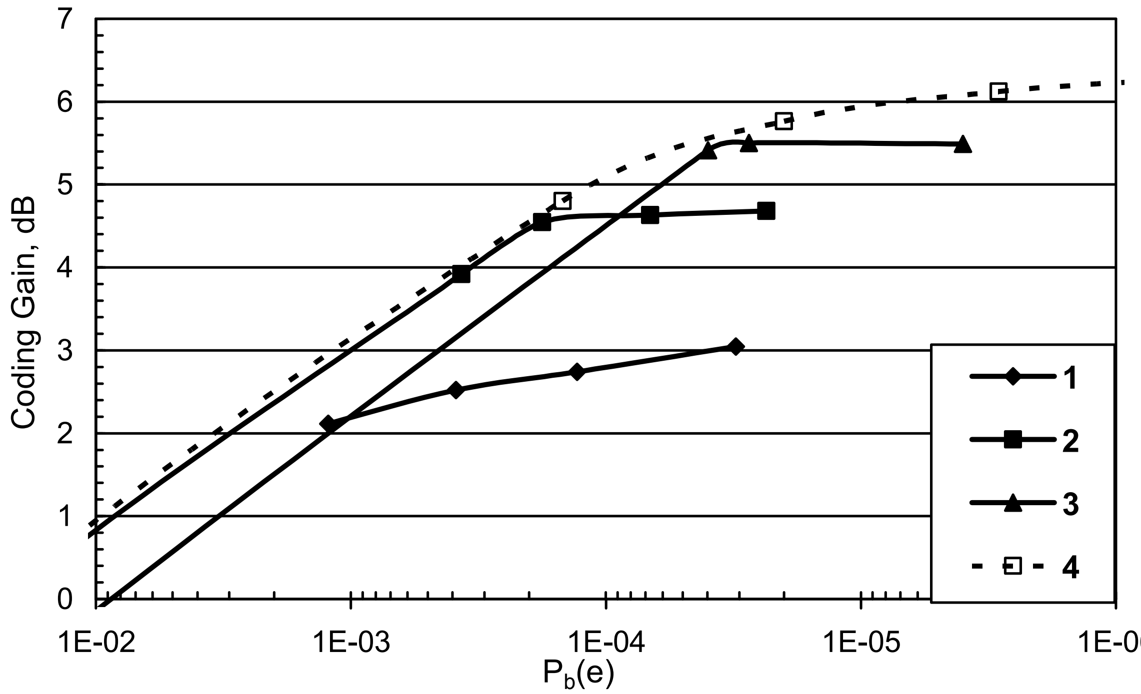

Fig.6

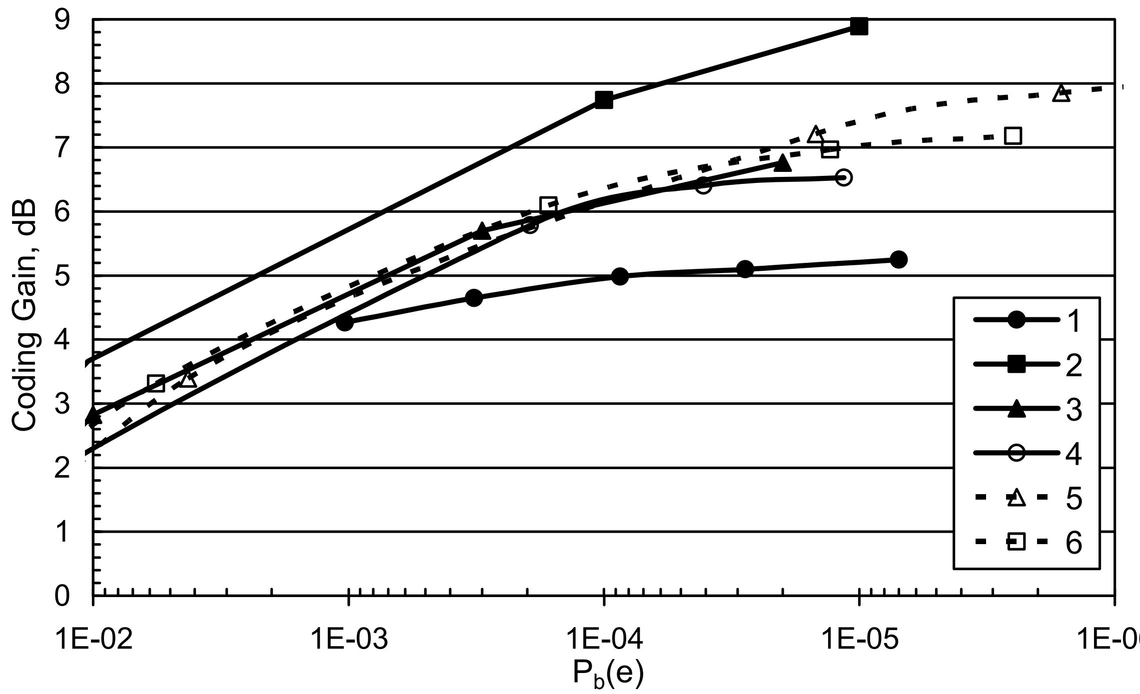

Fig.7

Described results are for binary codes and for systems with binary phase-shist keying. On the basis of the most important property of MTD solution convergence to the optimal one with all changes of decoded symbols [6] non-binary MTD are realized too, providing the characteristics close to or a bit higher than at Reed-Solomon codes. Analogically, that MTD provides high characteristics in channels with erasures and in systems with multi-position modulation. All these results make them an universe method for a simple achievement of high level of noise-proving of messages in systems of wide sphere of application. In this case the complexity of MTD realization is the same as for the simplest threshold decoder.

Conclusion. Nowadays technology capabilities allow to create hard–soft decoder's version even for high-speed channels, that will be able to do rather big quantity of operations with an every decoding symbol. It gives an opportunity to build up even more and more comprehensive code construction. Though algorithms, that use non-rationally the calculation resources, still yield to the much more simple ones, but solving the problem of decoding very effectively.

It is doubtless, that problems of complexity of realization of coding will do exist in the nearest future, and demands for simpler decoder realizations would be even more urgent because of the growth informational exchange speed. The cheapest in all variants of realization would be the algorithms providing only very simple, uniform and fast operations. MTD rather meets these demands. And correspondence of its abilities to the characteristics of the most complex algorithms makes MTD even more attractive.

Certainly, if it is necessary to achieve the levels of CG, that can be compared with the best turbo codes, sizes of memory of MTD and the number of providing by it operations of adding would increase really. But even in this case the complexity of its realizations will be, obviously, quite small. Though the necessity of decoder realization for communication equipment with energy only for a few tenths parts of dB, i.e. in 2–4% is less than possible limit, demands for deeper technological research and justification. Seems these questions become really urgent in a few years later, not earlier.

So in the period after publishing of the article [1] there were created conditions to achieve the levels of energy, quite close to theoretical limits for satellite channels and space communication. High characteristics could be provided with the help of some methods. Though their complexity with comparable CG levels shows that algorithms on the basis of MTD are close in number of operations to the simplest decoder of the threshold type. That is why in absolute majority of the cases MTD is the most preferable algorithm of decoding in high speed communication systems.

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