Effective Multithreshold Decoding Algorithms for Wireless Communication Channels

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***Abstract*— Multithreshold decoding algorithms for self-orthogonal error-correction codes have been considered. Analytical estimations of their efficiency in the uncorrelated Rayleigh and Rician channels have been represented. Results of the computer simulation have been obtained. Possibilities to apply multithreshold decoders in combination with such technologies of the efficiency increase of data transmission systems in wireless fading channels as OFDM, MIMO, precoding are considered. The paper has shown that application of precoding under such conditions ensures a significant improvement of coding gain of the data transmission system.**

***Index Terms*— error-correcting coding, self-orthogonal code, multithreshold decoder, coding gain, fading channel**

I. INTRODUCTION

Error-correction coding are applied in communication systems widely. At present there are a lot of codes and algorithms of their decoding, for example, Bose-Chaudhuri-Hocquenghem block codes, Reed-Solomon codes, convolutional codes, Low-Density Parity-Check (LDPC) codes and turbo codes. New and very effective solution of the low complexity decoding under simultaneous high coding gain on the base of multithreshold decoders (MTD) for self-orthogonal codes (SOC) has been suggested by Russian specialists [1].

At present MTD characteristics are well investigated for channels with independent errors where such method ensure near to optimal decoding (OD) even for very long codes only with linear implementation complexity [1, 2, 3]. For such channels a range of methods to improve MTD efficiency is known [1, 2, 4]. Also interesting results have been obtained for symbol MTD [4, 5]. Results of theoretical and experimental researches show MTD decoders require hundreds time less additive equivalent operation in comparison with known decoders for turbo and LDPC codes at similar coding gain. So MTD is very interesting for implementation in high rate communication and data storage systems [3, 4, 6, 7]. At the same time according to the coding theory, MTD efficiency in Gaussian channels can be slightly improved. Besides, future communication systems operate under more sophisticated conditions occurred due to the multipath signal propagation, Doppler shift and other reasons. As a result, errors occurred in channels are grouped into packages. Under such conditions, effect of the coding application is higher than in channels with independent errors since here in some cases it is impossible to decrease a bit error probability only by increase of the transmitter power [8]. Consequently, development of algorithms for increase of the MTD efficiency in channels with grouped errors (***that is the aim of the present paper***) will allow increasing an coding gain which can be used for improvement of technical characteristics of data transmission systems and significantly expand an application area of such method. Besides, it is very important to keep and only insignificantly increase a sophistication of the initial multithreshold decoder realization because only the simplest methods of error correction can ensure presently required decoding rate about tens Gb/s.

II. MULTITHRESHOLD DECODERS

Multithreshold decoders are used for decoding of block or convolutional self-orthogonal codes. Main principles of MTD operation used for decoding of block SOC with code rate 1/2 and length 26 bits set by the generator polynomial $g(x) = 1 + x^1 + x^4 + x^6$ are demonstrated by the scheme shown in Fig. 1 [1]. It should be noted that MTD composition includes registers, half-adders and a threshold element summing its inputs and comparing an obtained amount with the threshold. It makes MTD to be the simplest device for realization which can ensure maximum possible decoding rate.

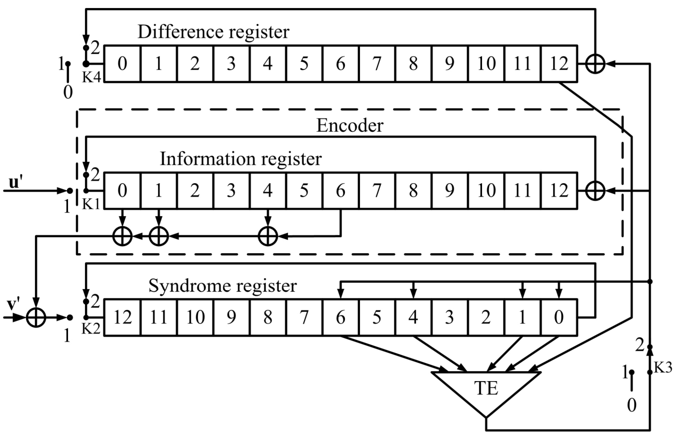

Fig. 1. MTD circuit of the block SOC

Under MTD operation in the binary symmetric channel, function L_j is calculated for the current decoding symbol u_j at the threshold element. This function depends on syndrome elements s_{jk} and corresponding element of the difference register d_j:

$$L_j = \sum_{s_{jk} \in \{S_j\}} s_{jk} + d_j \ , \qquad (1)$$

where $\{S_j\}$ – a set of checks (syndrome elements) in relation to the error e_j in the decoding symbol u_j. When the function exceeds some threshold value, change of the decoding symbol, corresponding element of the difference register and syndrome elements participating in calculation is executed.

Under MTD operation in the Gaussian channel, function L_j is also calculated for the current symbol u_j. And syndrome elements and element of the difference register are summed with some coefficients reflecting their reliability:

$$L_j = \sum_{s_{j_k} \in \{S_j\}} (2s_{j_k} - 1)w_{j_k} + (2d_j - 1)w_j , \qquad (2)$$

where w_{j_k} – a reliability of the check s_{j_k}; w_j – a reliability of the accepted symbol u_j. For example, an absolute value of likelihood ratio logarithm can be used as estimations of reliability of symbols accepted from the channel.

Papers [1, 2] show that decision of MTD converges to the OD decision under each change of the decoding symbol. This is because total weight of the syndrome and difference registers decreases under change of the symbol. Thus transfer to the maximum likelihood codeword may be done. However, due to occurrence of the error propagation effect [1, 2], process of transfer from one codeword to another that is more likely, can be stopped before MTD reaches a solution of the OD. Such effect is that a large number of errors made by the decoder enter the syndrome through branches of feedback after the decoder error occurrence. These errors prevent from correct decoding of informational symbols at following iterations. This leads to the fact that probability of the second error increases and error burst appears. Consequently, it is necessary to use codes maximally resistant to error propagation [1, 2] in order to approximate a probability of MTD error correction to the OD one. The error propagation estimation described in [1] can be used under code construction.

III. Analytical estimation of the MTD error probability in Rayleigh and Rician channels

The simplest models of fading channels are models of channels with Rayleigh and Rician fading which occur under presence of a multipath signal propagation. At the same time there is no line of sight between a transmitter and receiver in the Rayleigh channel, but it is present in the Rician channel.

Let's obtain a lower bound of the MTD error probability. For simplicity, we assume that BPSK or QPSK modulation and hard decision demodulator are used. Let's note that for MTD the lower bound of error probability in the binary symmetric channel is determined by error probability of the OD calculated according to expression [1, 2]:

$$P_b = \sum_{i=(d+1)/2}^{d} C_d^i \ p^i \left(1-p\right)^{d-i}, \qquad (3)$$

where d – a code distance of the used SOC (assumed as odd); p – probability of an error in the channel.

For the noncorrelated Rayleigh channel, expression of the error probability is known [9]

$$p = \frac{1}{2}\left(1 - \sqrt{\frac{E_S / N_0}{1 + E_S / N_0}}\right), \qquad (4)$$

where E_s/N_0 – a signal to noise ratio. Such estimation is correct for BPSK or QPSK modulation and Doppler shift $F_d = 0$. In the case of the channel with Rician fading error probability according to [9] is determined as

$$p = 2Q\left(\sqrt{\frac{2kE_S / N_0}{k + E_S / N_0}}\right), \qquad (5)$$

where k – a Rician coefficient; $Q(x)$ is determined as

$$Q\left(x\right) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt \ . \qquad (6)$$

Taking into account that MTD can operate almost as OD, we put (4) into (3) and obtain the lower bound of the bit error probability for MTD in the Rayleigh channel:

$$P_b = \sum_{i=(d+1)/2}^{d} \left(C_d^i \frac{1}{2}\left(1 - \sqrt{\frac{E_S / N_0}{1 + E_S / N_0}}\right)^i \cdot \right. \\ \left. \cdot \left(1 - \frac{1}{2}\left(1 - \sqrt{\frac{E_S / N_0}{1 + E_S / N_0}}\right)\right)^{d-i} \right). \qquad (7)$$

Similarly, bit error probability in the Rician channel is determined as

$$P_b = \sum_{i=(d+1)/2}^{d} \left(C_d^i Q\left[\sqrt{\frac{2kE_S / N_0}{k + E_S / N_0}}\right]^i \cdot \right. \\ \left. \cdot \left(1 - Q\left[\sqrt{\frac{2kE_S / N_0}{k + E_S / N_0}}\right]\right)^{d-i} \right). \qquad (8)$$

Let's note that expressions in such form allow obtaining an estimation of the MTD error probability under usage of SOC with the odd code distance.

In Fig. 2 curves *5* and *6* show estimation of the error probability in the noncorrelated Rayleigh channel and error probability after MTD decoder for SOC with a minimum code distance $d = 9$. Let's note that losses about 1,5 dB are observed in comparison with the Gaussian channel. In the same figure curves *1* and *2* show estimation of the channel and MTD error probability obtained by means of computer simulation using a block SOC with length 20000 bits, code rate $R = 1/2$ and code distance $d = 9$ and MTD with 15 decoding iterations. Let's note that estimation of the channel error probability well conforms to the analytical one that allows using the latest one in obtained formulas. Besides, we can see that obtained analytical estimation of the decoder error probability under usage of MTD in the noncorrelated Rayleigh channel is enough correct for the area of MTD effective operation. Similar results for the Rician channel under $k = 5$ are shown in Fig. 2 by curves *7, 8* for analytical estimations and curves *3, 4* for results of the simulation. And in this case obtained estimation is good for the area of MTD effective operation.

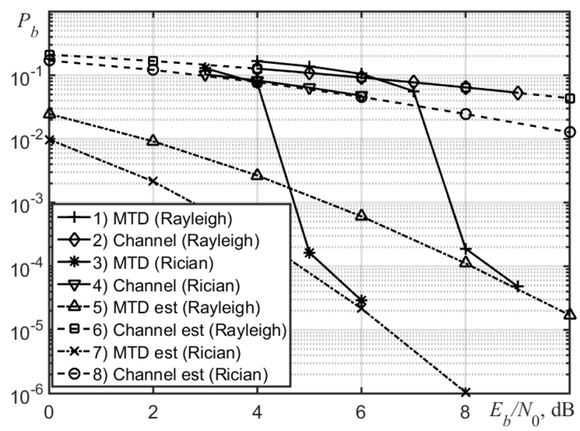

Fig. 2. Analytical estimations of the error probability and results of the simulation for MTD in Rayleigh and Rician channels

Let's note that it is not possible to obtain analytical estimations with accepted accuracy under present of correlated fading or usage of the modulation of a higher order for larger noise. So, under such conditions computer simulation should be used for estimation of the MTD efficiency.

IV. Results of the Simulation of MTD in Rayleigh and Rician Channels

In Fig. 3 curves *2, 4* and *6* show results of the simulation for Rayleigh and Rician fading under Doppler frequency 100 and 150 Hz and usage of BPSK modulation and hard decision demodulator. Here MTD was used for the code similar to the code presented in the Fig. 2. We note that presence of correlated fading significantly (for Rayleigh fading – up to 6 dB) worsens an operation signal to noise ratio. This is because in such case a code with parallel concatenation was used and data from the coder are delivered into the modulator "by lines" without usage of an interleaver. As a result under occurrence of fading, the whole branches with greater check dimension could be distorted. That led to appearance of blocks with a significant number of errors after decoding. Additional interleaver should be used, that is not always convenient, or internal interlacing should be organized giving data from the coder "by columns" in order to improve the MTD efficiency. In Fig. 3 curves *1, 3* and *5* show results of the simulation corresponding to the suggested parallel bit transmission. Schedule review has shown that in the case of bit transmission "by columns" in the channel with Rician fading, decrease of the decoding error probability in comparison with the original variant, for example, under signal to noise ratio 7 dB under $k = 5$ and $F_d = 150$, becomes greater two decimal exponents. Gain is also observed under other simulation conditions. This is because under parallel transmission, errors caused by signal fading become diversed by different branches. Besides, branches with greater check dimension are distorted much less and MTD becomes capable to ensure less decoding error probability.

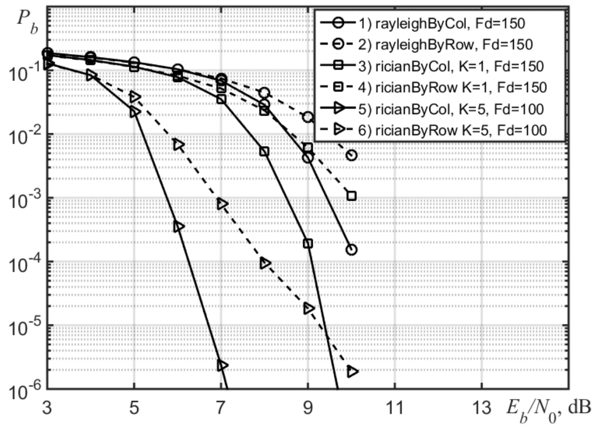

Fig. 3. MTD characteristics in the fading channels under successive and parallel bit transmission

V. MTD Operation in Wireless Communication Systems with Precoding

We should note that mentioned cases of the MTD application allow successfully error correction if there is no any intersymbol interference. If intersymbol interference is present, it is necessary to use additional facilitates as orthogonal frequency division multiplexing (OFDM) or others. Besides, analysis of results presented in [10] has shown that decrease of the error probability is slower in channels with fading under increase of the signal to noise ratio in comparison with the Gaussian channel. This is because under long fading even under high signal to noise ratio, a large error packages appears at the decoder input. The decoder cannot correct them. Technology of the spatial diversity is often used to decrease influence of such effect when several transmitting and receiving antennas are used (Multiple Input Multiple Output – MIMO). Under correct development of such system we can consider that resulting sub channels turn out to be independent. Consequently, probability of the fact that all channels are simultaneously exposed to fading will be much less in comparison with the single-channel system. It allows significantly decreasing a reliability of the data transmission.

We note that under usage of MIMO there is an additional possibility to use of Space-Time Coding (STC) [11] – coding reflecting transmitting symbols of the signal constellation to transmitting antennas which usage allows significantly improving characteristics of the system using MTD [12].

Also we note that previously obtained results supposed that the receiving side exactly knows a channel state, i.e. multiplicative noise component is known for each transmitting channel. In practice pilot symbols added to the transmitting channel are used for estimation of the channel. The demodulator used this exact information under estimation of signals received from the channel. In some cases such information is also known at the transmitting side (for example, in the case when channel is changed enough slowly, transmission of this information can be organized by the reverse channel). The transmitter can use information of the channel state for redistribution of the signal energy by spatial channels by loading "good" channels and emptying "bad" channels. Such actions are fulfilled by so-called precoding of the transmitting signal. Let's consider precoding in details.

In the case of usage of MIMO transmitter architecture when each antenna transmittes its own symbol, the channel model is described by expression

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{7}$$

where $\mathbf{x} \in \mathrm{C}^M$ – a transmitting vector from M complex values; $\mathbf{n} \in \mathrm{C}^N$ – a vector of the uncorrelated complex Gaussian noise; $\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_N) \in \mathrm{C}^{M \times N}$ – a complex matrix of the channel; M – a number of transmitting antennas; N – a number of receiving antennas. We note that values $h_{i,j}$ are random, but for the mentioned variant they are known by the transmitting and receiving sides.

For such channel model, vector of informational symbols $\mathbf{s} \in \mathrm{C}^L$ from some alphabet is precoded by the linear operation

$$\mathbf{x} = \mathbf{B}\mathbf{s}, \tag{8}$$

where $\mathbf{B} \in \mathrm{C}^{M \times L}$ – a complex matrix of the precoder; $L \le \mathrm{rank}(\mathbf{H})$ – a number of active channels.

Optimal MMSE receiver for such channel and precoder is represented as a Wiener filter with matrix

$$\mathbf{G} = \mathbf{B}^*\mathbf{H}^*(\mathbf{H}\mathbf{B}\mathbf{B}^*\mathbf{H}^* + \sigma_n^2 \mathbf{I}_N)^{-1}, \tag{9}$$

where ()* – a complex conjugation; σ_n^2 – a variance of the Gaussian noise; $\mathbf{I}_N$ – an identity matrix with dimensions NxN.

Such filter provides an optimal estimation of transmitted symbols at the output

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{y}. \tag{10}$$

There are a great number of various algorithms of precoding including different in complexity of the optimal receiver. Some interesting precoders optimal by some criterion which receiver has a linear complexity are suggested in [13].

The first of these algorithms minimizes a total root-mean-square error at the receiver output. For such precoder matrix of precoding has a form

$$\mathbf{B} = \tilde{\mathbf{V}}\mathbf{\Phi}^{1/2}, \tag{11}$$

where $\tilde{\mathbf{V}} \in \mathrm{C}^{M \times L}$ – a complex matrix made of the first L columns of the unitary matrix contained in the singular decomposition of the symmetric matrix $\mathbf{HH}^*$ where all eigenvalues λ_{nn} of the diagonal matrix $\Lambda = \mathrm{diag}(\lambda_{11}, \lambda_{22}, ..., \lambda_{NN})$ are negative and ordered by descending; $\mathbf{\Phi} \in \mathrm{R}^{L \times L}$ – a diagonal matrix consisting of L negative numbers

$$\phi_{ii} = \left(\frac{P_0 + \sigma_n^2 \sum_{n=1}^{\bar{L}} \lambda_{nn}^{-1}}{\sum_{n=1}^{\bar{L}} \lambda_{nn}^{-1/2}} \lambda_{ii}^{-1/2} - \frac{\sigma_n^2}{\lambda_{ii}} \right)^+, \ \iota=1,...,\Lambda, \tag{12}$$

where $(x)^+ = \max(x, 0)$; $\bar{L} \le L$ is chosen as $\phi_{nn} > 0$ for $n \in [1, \bar{L}]$ and $\phi_{nn} = 0$ for all other n; P_0 – a mean power of the transmitting signal.

The second precoder maximizes transmitted information between tranmitting and receiving data. As distinct from the first variant here elements of the matrix $\mathbf{\Phi} \in \mathrm{R}^{L \times L}$ are determined by following

$$\phi_{ii} = \left(\frac{P_0 + \sigma_n^2 \sum_{n=1}^{\bar{L}} \lambda_{nn}^{-1}}{\bar{L}} \lambda_{ii}^{-1} - \frac{\sigma_n^2}{\lambda_{ii}} \right)^+. \ \iota=1,...,\Lambda. \tag{11}$$

Then we consider an efficiency of the application of these algorithms together with MTD. Under simulation, MTD is used with 30 decoding iterations for the designed SOC with code rate $R = 8/16$, code distance 17 and length 43200 bits. OFDM with 1024 subcarriers is applied together with MTD. Guard interval was 1/16 of the OFDM symbol length. Ordinary QPSK was used as modulation. Spatial Channel Model of the type Urban micro was used under acquisition of results. Maximum Doppler frequency F_d was equal to 0. Figure 4 shows MTD characteristics under above mentioned conditions under usage of a various number of transmitting and receiving antennas and various precoding algorithms.

In this figure, a demodulator formed only hard decisions under acquisition of curves *1* and *3*, and it estimates reliability of decisions under acquisition of other curves. Let's note that using of soft decision demodulator ensures an increase of the coding gain up to 2 dB in comparison with usage of hard decisions. Also we would like to note that 2x2 variant by energy becomes worse than a single-channel one in 3 dB, but spectral efficiency of the system becomes twice better. Usage of precoding together with two transmitting antennas decreases a loss by energy up to 1 dB. Interesting results are observed for four transmitting antennas. Here under usage of the second precoder results are significantly improved. It becomes possible due to the fact that "bad" spatial channels become disconnected more efficiently in comparison with the first precoder (for example, under signal to noise ratio 5 dB, 2.5 of 4 spatial channels are averagely used, and for the first precoder – 3.5 channels). As a results energy characteristics become better than a single-channel variant more than in 3.5 dB under aproximately 2.5 times better spectral efficiency.

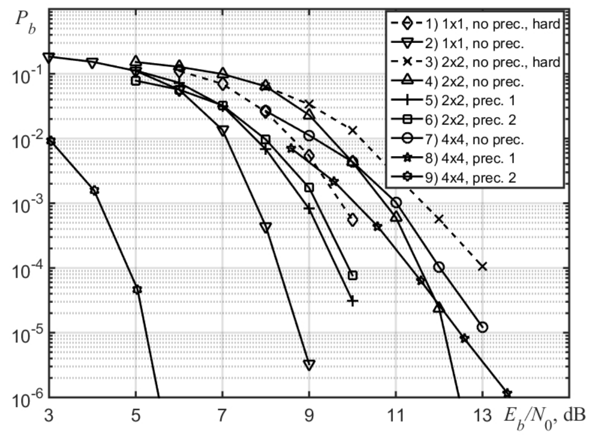

Fig. 4. MTD characteristics for various numbers of receiving and transmitting antennas and various precoding algorithms

So, obtained results allow stating that due to application of MIMO and precoding it is possible to improve significantly MTD efficiency in multipath fading channels in comparison with a single-channel variant. We note that results obtained in this section are acquired within research and development works executed under support of the Russian Scientific Foundation (project №14-19-01263).

VI. Conclusion

The paper has obtained new results of the MTD efficiency research in a range of typical multipath models of communication channels with fading. Recommendations for the better matching of coder and channel, uncorrelated Rayleigh and Rician channels have been given for these channels. The lowest estimations of the multithreshold decoding error probability have been obtained. Research of the precoding application efficiency together with multithreshold decoders and MIMO technology has been executed.

Analysis of the results represented in the paper has shown that MTD providing high efficiency in Gaussian channels become capable to error correction under even more sophisticated conditions of application. Besides, effect of the error-correcting coding application exceeds an effect of the coding application in the Gaussian channel in many times under fading channels.

Acknowledgment

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